A Minimal Symbolic Calculus:
Structural Operators and Their Phenomenological Correlates

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Abstract

We introduce a minimal formal system generated by four primitive operators: $\square$, $\triangle$, $\circ$, and $\ldots$. These operators are defined purely mathematically as acting on abstract domains. We establish their algebraic properties, interaction rules, and expressive limits. In a second stage, we provide a phenomenological association, interpreting these operators as structural invariants of lived experience without reducing mathematics to psychology. The result is a bifurcated but coherent account of symbol, form, and meaning.

1 Introduction

Minimality has long been a guiding principle in both mathematics and philosophy. In this paper we investigate whether a small set of abstract operators can generate a rich symbolic calculus. We restrict ourselves to four primitives and ask two questions:

1. What formal structure do these operators generate?
2. What, if anything, corresponds to this structure in experience?

The first question is mathematical; the second is phenomenological. They are treated separately.

2 Formal Setting

Definition 1 (Domain)**.** *Let X be a nonempty set. A* domain *is any subset $D \subseteq X$.*

Definition 2 (Symbolic Calculus)**.** *A symbolic calculus $\mathcal{C}$ is a tuple*

$$\mathcal{C} = (X, \mathcal{O})$$

where X is a domain and $\mathcal{O}$ is a finite set of operators on X.

We take

$$\mathcal{O} = \{\square, \triangle, \circ, \ldots\}.$$

3 The Square Operator

Definition 3 (Stability Operator). *The operator* $\Box : \mathcal{P}(X) \to \mathcal{P}(X)$ *satisfies:*

$$\Box(D) \subseteq D \quad \textit{and} \quad \Box(D) = \Box(\Box(D)).$$

Lemma 1 (Idempotence). $\Box$ *is idempotent.*

Proof. By definition, $\Box(\Box(D)) = \Box(D)$. ∎

Proposition 1 (Fixed Points). *A domain* D *is stable if and only if* $\Box(D) = D$.

4 The Triangle Operator

Definition 4 (Transform Operator). *The operator* $\triangle : \mathcal{P}(X) \to \mathcal{P}(X)$ *satisfies:*

$$\triangle(D) \not\subseteq D \quad \textit{and} \quad \triangle(D_1 \cup D_2) \neq \triangle(D_1) \cup \triangle(D_2).$$

Lemma 2 (Nonlinearity). $\triangle$ *is non-additive.*

Proof. Transformation introduces interaction terms not present in the union of independent domains. ∎

Proposition 2 (Generativity). *Repeated application of* $\triangle$ *produces an unbounded sequence of distinct domains.*

5 The Circle Operator

Definition 5 (Closure Operator). *The operator* $\circ : \mathcal{P}(X) \to \mathcal{P}(X)$ *satisfies:*

$$D \subseteq \circ(D) \quad \textit{and} \quad \circ(\circ(D)) = \circ(D).$$

Lemma 3 (Recursion). $\circ$ *introduces self-reference.*

Proof. Closure requires the inclusion of the domain's own transformations. ∎

Proposition 3 (Equilibrium). *If* $\circ(D) = D$*, then* D *is recursively complete.*

6 The Ellipsis Operator

Definition 6 (Indefinite Extension). *The operator* $\cdots : \mathcal{P}(X) \to \mathcal{P}(X)$ *is defined only as a limit:*

$$\ldots(D) = \lim_{n\to\infty} F_n(D),$$

where no finite F_n *yields a terminal domain.*

Lemma 4 (Non-Computability)**.** *... is not computable in finite time.*

Proposition 4 (Incompleteness)**.** *No finite composition of* $\square$*,* $\triangle$*, and* $\circ$ *can eliminate ....*

7 Algebraic Structure

Theorem 1 (Minimal Completeness)**.** *The calculus* $\mathcal{C}$ *is expressively complete but formally incomplete.*

Proof. $\square$ ensures persistence, $\triangle$ ensures novelty, $\circ$ ensures coherence, and ... prevents closure. Removing any operator collapses one of these capacities. $\square$

8 Phenomenological Association

We now introduce interpretation without altering the formal system.

Definition 7 (Phenomenological Correlate)**.** *A phenomenological correlate is an invariant structure of experience that mirrors a formal operator without being reducible to it.*

8.1 Stability and Retention

$\square$ corresponds to retention: the persistence of identity across temporal flow. Without it, no object remains the same from moment to moment.

8.2 Transformation and Protention

$\triangle$ corresponds to directed anticipation. Experience is not static; it moves toward what is not yet given.

8.3 Closure and Horizon

$\circ$ corresponds to the experiential horizon: the sense that experience is whole, even though only parts are given at any time.

8.4 Indeterminacy and Openness

... corresponds to the excess of experience over articulation. Every act of meaning leaves a remainder.

9 Final Theorem

Theorem 2 (Structural Isomorphism)**.** *The formal calculus* $\mathcal{C}$ *is structurally isomorphic to the minimal conditions of coherent experience.*

Proof. Both systems require persistence, transformation, coherence, and openness. The mapping preserves relations without collapsing domains. □

10 Conclusion

The symbolic universe does not arise from representation alone. It arises from structure.

Mathematics reveals the structure. Phenomenology reveals that we live inside it.

$\square \quad \triangle \quad \circ \quad \ldots$